```
+ M2.exe --no-readline --print-width 157
Macaulay2, version 1.6
with packages: ConwayPolynomials, Elimination, IntegralClosure, LLLBases, PrimaryDecomposition, R
eesAlgebra, TangentCone

i1 : input"nm2wedges.m2"

ii2 : printWidth = 97;

ii3 : cG = collectGarbage;

ii4 : run"date";
Wed, Sep 18, 2013  6:07:07 PM

ii5 : ------------------------------------------------------------------------------
      --We look for the minimal primes of the ideal generated by the principal t-minors of a size
 n matrix of indeterminates.
      ------------------------------------------------------------------------------
      needsPackage"Depth";

ii6 : needsPackage("MonomialAlgebras",Configuration=>{"Use4ti2"=>true});
--loading configuration for package "MonomialAlgebras" from file /home/Audrey/.Macaulay2/init-Mon
omialAlgebras.m2
--loading configuration for package "FourTiTwo" from file /home/Audrey/.Macaulay2/init-FourTiTwo.
m2

ii7 : needsPackage"Binomials";

ii8 : ptM = method(); --computes the ideal generated by the principal t-minors

ii9 :   ptM(ZZ,Matrix) := (t,M)->(
                ideal(
                        m := min(numrows M,numcols M);
                        s := subsets(m,t);
                        (0..#s-1)/(i->det(submatrix(M,s_i,s_i),Strategy=>Bareiss))
                        )
                );

ii10 : -----
       --User prompts:
       <<"Default ground field: "<<describe kk;
                     ZZ
Default ground field: ---
                     101
ii11 :                                          {* (if value(read"Use? (y/n) ")!=y then
                                                        kk=value read"Specify the ground field: "
);
                                                        *}
       n = value read"Specify matrix size: ";
Specify matrix size: 5

ii12 : t = n-2;

ii13 : r = n-1;

ii14 : -----
       S' = kk[u_1..u_r,v_1..v_r,w_(1,1)..w_(r,r),
        MonomialSize=>64];

ii15 : A = (
        bfu := matrix{toList(u_1..u_r)};
        matrix entries(1_S'||bfu)
        );

                5        4
oo15 : Matrix S'  <--- S'

ii16 : <<A<<endl;
| 1   0   0   0   |
| 0   1   0   0   |
| 0   0   1   0   |
| 0   0   0   1   |
| u_1 u_2 u_3 u_4 |

ii17 : B = (
        bfv := matrix transpose {toList(v_1..v_r)};
        1_S'|bfv
        );
```

```
                  4         5
oo17 : Matrix S'  <--- S'

ii18 : <<B<<endl;
| 1 0 0 0 v_1 |
| 0 1 0 0 v_2 |
| 0 0 1 0 v_3 |
| 0 0 0 1 v_4 |

ii19 : whatDies = (
        W' := (
                W := matrix transpose entries genericMatrix(S',w_(1,1),r,r);
                sub(W,toList(1..r)/(i->w_(i,i)=>0))
                ); --exteriorPower(t,W);
        <<"W' = "<<W'<<endl;
        Delta := det W';
        M' := (
                A' := exteriorPower(t,A);
                <<"A' = "<<A'<<endl;
                B' := exteriorPower(t,B);
                <<"B' = "<<B'<<endl;
                A'*W'*B'
                );
        zer := ideal(flatten entries compress gens ptM(1,M'));
        saturate(zer,Delta)
        );
W' = | 0       w_(1,2) w_(1,3) w_(1,4) |
     | w_(2,1) 0       w_(2,3) w_(2,4) |
     | w_(3,1) w_(3,2) 0       w_(3,4) |
     | w_(4,1) w_(4,2) w_(4,3) 0       |
A' = | 1    0    0    0    |
     | 0    1    0    0    |
     | 0    0    1    0    |
     | 0    0    0    1    |
     | u_3  u_4  0    0    |
     | -u_2 0    u_4  0    |
     | u_1  0    0    u_4  |
     | 0    -u_2 -u_3 0    |
     | 0    u_1  0    -u_3 |
     | 0    0    u_1  u_2  |
B' = | 1 0 0 0 v_3 -v_2 v_1 0    0    0   |
     | 0 1 0 0 v_4 0    0   -v_2 v_1  0   |
     | 0 0 1 0 0   v_4  0   -v_3 0    v_1 |
     | 0 0 0 1 0   0    v_4 0    -v_3 v_2 |

oo19 : Ideal of S'

ii20 : <<"What are the minimal primes of "<<endl<<matrix transpose sort entries gens whatDies<<"?
"<<endl;
What are the minimal primes of
| u_1v_2w_(3,4)+u_2v_1w_(4,3)                                           |?
| u_1v_3w_(2,4)+u_3v_1w_(4,2)                                           |
| u_2v_3w_(2,3)+u_3v_2w_(3,2)                                           |
| u_1v_4w_(1,4)+u_4v_1w_(4,1)                                           |
| u_2v_4w_(1,3)+u_4v_2w_(3,1)                                           |
| u_3v_4w_(1,2)+u_4v_3w_(2,1)                                           |
| u_1u_2u_3v_4w_(2,3)w_(3,4)w_(4,2)+u_1u_2u_3v_4w_(2,4)w_(3,2)w_(4,3) |
| u_4v_1v_2v_3w_(2,3)w_(3,4)w_(4,2)+u_4v_1v_2v_3w_(2,4)w_(3,2)w_(4,3) |
| u_3v_1v_2v_4w_(1,3)w_(3,4)w_(4,1)+u_3v_1v_2v_4w_(1,4)w_(3,1)w_(4,3) |
| u_1u_2u_4v_3w_(1,3)w_(3,4)w_(4,1)+u_1u_2u_4v_3w_(1,4)w_(3,1)w_(4,3) |
| u_2v_1v_3v_4w_(1,2)w_(2,4)w_(4,1)+u_2v_1v_3v_4w_(1,4)w_(2,1)w_(4,2) |
| u_1u_3u_4v_2w_(1,2)w_(2,4)w_(4,1)+u_1u_3u_4v_2w_(1,4)w_(2,1)w_(4,2) |
| u_1v_2v_3v_4w_(1,2)w_(2,3)w_(3,1)+u_1v_2v_3v_4w_(1,3)w_(2,1)w_(3,2) |
| u_2u_3u_4v_1w_(1,2)w_(2,3)w_(3,1)+u_2u_3u_4v_1w_(1,3)w_(2,1)w_(3,2) |

ii21 : mP = binomialMinimalPrimes whatDies;

ii22 : apply(mP,P->(
        <<endl<<matrix transpose sort entries gens P;
        <<endl<<"codim = "<<codim P;
        <<endl<<"depth = "<<depth(P,S')<<endl
        ));
| w_(2,3)w_(3,4)w_(4,2)+w_(2,4)w_(3,2)w_(4,3)       |
| w_(1,3)w_(3,4)w_(4,1)+w_(1,4)w_(3,1)w_(4,3)       |
| w_(1,2)w_(2,4)w_(4,1)+w_(1,4)w_(2,1)w_(4,2)       |
| u_1v_2w_(3,4)+u_2v_1w_(4,3)                       |
| w_(1,2)w_(2,3)w_(3,1)+w_(1,3)w_(2,1)w_(3,2)       |
| u_1v_3w_(2,4)+u_3v_1w_(4,2)                       |
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```
| u_2v_3w_(2,3)+u_3v_2w_(3,2)                                    |
| u_1v_4w_(1,4)+u_4v_1w_(4,1)                                    |
| u_2v_4w_(1,3)+u_4v_2w_(3,1)                                    |
| u_3v_4w_(1,2)+u_4v_3w_(2,1)                                    |
| w_(1,3)w_(2,1)w_(3,4)w_(4,2)-w_(1,2)w_(2,4)w_(3,1)w_(4,3)      |
| u_3v_2w_(3,4)w_(4,2)-u_2v_3w_(2,4)w_(4,3)                      |
| w_(1,2)w_(2,3)w_(3,4)w_(4,1)-w_(1,4)w_(2,1)w_(3,2)w_(4,3)      |
| u_4v_2w_(3,4)w_(4,1)-u_2v_4w_(1,4)w_(4,3)                      |
| w_(1,3)w_(2,4)w_(3,2)w_(4,1)-w_(1,4)w_(2,3)w_(3,1)w_(4,2)      |
| u_4v_3w_(2,4)w_(4,1)-u_3v_4w_(1,4)w_(4,2)                      |
| u_1v_3w_(2,3)w_(3,4)-u_3v_1w_(3,2)w_(4,3)                      |
| u_1v_4w_(1,3)w_(3,4)-u_4v_1w_(3,1)w_(4,3)                      |
| u_1v_2w_(2,4)w_(3,2)-u_2v_1w_(2,3)w_(4,2)                      |
| u_4v_3w_(2,3)w_(3,1)-u_3v_4w_(1,3)w_(3,2)                      |
| u_1v_2w_(1,4)w_(3,1)-u_2v_1w_(1,3)w_(4,1)                      |
| u_1v_4w_(1,2)w_(2,4)-u_4v_1w_(2,1)w_(4,2)                      |
| u_2v_4w_(1,2)w_(2,3)-u_4v_2w_(2,1)w_(3,2)                      |
| u_1v_3w_(1,4)w_(2,1)-u_3v_1w_(1,2)w_(4,1)                      |
| u_2v_3w_(1,3)w_(2,1)-u_3v_2w_(1,2)w_(3,1)                      |
| u_4v_2w_(2,1)w_(3,4)w_(4,2)+u_2v_4w_(1,2)w_(2,4)w_(4,3)        |
| u_3v_4w_(1,3)w_(3,4)w_(4,2)+u_4v_3w_(2,4)w_(3,1)w_(4,3)        |
| u_4v_3w_(2,3)w_(3,4)w_(4,1)+u_3v_4w_(1,4)w_(3,2)w_(4,3)        |
| u_3v_2w_(1,2)w_(3,4)w_(4,1)+u_2v_3w_(1,4)w_(2,1)w_(4,3)        |
| u_4v_2w_(2,4)w_(3,2)w_(4,1)+u_2v_4w_(1,4)w_(2,3)w_(4,2)        |
| u_2v_3w_(1,3)w_(2,4)w_(4,1)+u_3v_2w_(1,4)w_(3,1)w_(4,2)        |
| u_1v_4w_(1,2)w_(2,3)w_(3,4)+u_4v_1w_(2,1)w_(3,2)w_(4,3)        |
| u_1v_3w_(1,3)w_(2,1)w_(3,4)+u_3v_1w_(1,2)w_(3,1)w_(4,3)        |
| u_1v_4w_(1,3)w_(2,4)w_(3,2)+u_4v_1w_(2,3)w_(3,1)w_(4,2)        |
| u_1v_2w_(1,4)w_(2,1)w_(3,2)+u_2v_1w_(1,2)w_(2,3)w_(4,1)        |
| u_1v_2w_(1,2)w_(2,4)w_(3,1)+u_2v_1w_(1,3)w_(2,1)w_(4,2)        |
| u_1v_3w_(1,4)w_(2,3)w_(3,1)+u_3v_1w_(1,3)w_(3,2)w_(4,1)        |
codim = 6
depth = 6

| v_4 |
| v_3 |
| v_2 |
| v_1 |
codim = 4
depth = 4

| v_4                                                |
| u_4                                                |
| w_(2,3)w_(3,4)w_(4,2)+w_(2,4)w_(3,2)w_(4,3)        |
| u_1v_2w_(3,4)+u_2v_1w_(4,3)                        |
| u_1v_3w_(2,4)+u_3v_1w_(4,2)                        |
| u_2v_3w_(2,3)+u_3v_2w_(3,2)                        |
| u_3v_2w_(3,4)w_(4,2)-u_2v_3w_(2,4)w_(4,3)          |
| u_1v_3w_(2,3)w_(3,4)-u_3v_1w_(3,2)w_(4,3)          |
| u_1v_2w_(2,4)w_(3,2)-u_2v_1w_(2,3)w_(4,2)          |
codim = 5
depth = 5

| v_3                                                |
| u_3                                                |
| w_(1,3)w_(3,4)w_(4,1)+w_(1,4)w_(3,1)w_(4,3)        |
| u_1v_2w_(3,4)+u_2v_1w_(4,3)                        |
| u_1v_4w_(1,4)+u_4v_1w_(4,1)                        |
| u_2v_4w_(1,3)+u_4v_2w_(3,1)                        |
| u_4v_2w_(3,4)w_(4,1)-u_2v_4w_(1,4)w_(4,3)          |
| u_1v_4w_(1,3)w_(3,4)-u_4v_1w_(3,1)w_(4,3)          |
| u_1v_2w_(1,4)w_(3,1)-u_2v_1w_(1,3)w_(4,1)          |
codim = 5
depth = 5

| v_4                          |
| v_3                          |
| u_4                          |
| u_3                          |
| u_1v_2w_(3,4)+u_2v_1w_(4,3)  |
codim = 5
depth = 5

| v_2                                                |
| u_2                                                |
| w_(1,2)w_(2,4)w_(4,1)+w_(1,4)w_(2,1)w_(4,2)        |
| u_1v_3w_(2,4)+u_3v_1w_(4,2)                        |
| u_1v_4w_(1,4)+u_4v_1w_(4,1)                        |
| u_3v_4w_(1,2)+u_4v_3w_(2,1)                        |
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```
| u_4v_3w_(2,4)w_(4,1)-u_3v_4w_(1,4)w_(4,2)   |
| u_1v_4w_(1,2)w_(2,4)-u_4v_1w_(2,1)w_(4,2)   |
| u_1v_3w_(1,4)w_(2,1)-u_3v_1w_(1,2)w_(4,1)   |
codim = 5
depth = 5

| v_4                          |
| v_2                          |
| u_4                          |
| u_2                          |
| u_1v_3w_(2,4)+u_3v_1w_(4,2) |
codim = 5
depth = 5

| v_3                          |
| v_2                          |
| u_3                          |
| u_2                          |
| u_1v_4w_(1,4)+u_4v_1w_(4,1) |
codim = 5
depth = 5

| v_1                                          |
| u_1                                          |
| w_(1,2)w_(2,3)w_(3,1)+w_(1,3)w_(2,1)w_(3,2) |
| u_2v_3w_(2,3)+u_3v_2w_(3,2)                  |
| u_2v_4w_(1,3)+u_4v_2w_(3,1)                  |
| u_3v_4w_(1,2)+u_4v_3w_(2,1)                  |
| u_4v_3w_(2,3)w_(3,1)-u_3v_4w_(1,3)w_(3,2)   |
| u_2v_4w_(1,2)w_(2,3)-u_4v_2w_(2,1)w_(3,2)   |
| u_2v_3w_(1,3)w_(2,1)-u_3v_2w_(1,2)w_(3,1)   |
codim = 5
depth = 5

| v_4                          |
| v_1                          |
| u_4                          |
| u_1                          |
| u_2v_3w_(2,3)+u_3v_2w_(3,2) |
codim = 5
depth = 5

| v_3                          |
| v_1                          |
| u_3                          |
| u_1                          |
| u_2v_4w_(1,3)+u_4v_2w_(3,1) |
codim = 5
depth = 5

| v_2                          |
| v_1                          |
| u_2                          |
| u_1                          |
| u_3v_4w_(1,2)+u_4v_3w_(2,1) |
codim = 5
depth = 5

| u_4 |
| u_3 |
| u_2 |
| u_1 |
codim = 4
depth = 4


i24 :
```